LITERATURE CITED

1. V. P. Belyakov, *Cryogenic Engineering and Technology* [in Russian], Moscow (1982).
2. N. V. Filin and A. B. Bulanov, *Liquid Cryogenic Systems* [in Russian], Leningrad (1985).

THE HYDRODYNAMICS OF CERTAIN SURFACE PHENOMENA

V. B. Okhotskii

UDC 532.529

Based on wave theory, we have derived expressions to describe the hydrodynamics of certain surface phenomena. We have determined the conditions of their applicability.

In a number of branches of engineering, the course of physicochemical processes is governed by such superficial phenomena as wetting, threading, etc. The hydrodynamics of such processes is generally treated from the standpoint of viscous fluid flow [1]. On the basis of derived quantitative relationships one might assume that liquid motion is based on wave processes.

With the flow of an inviscid fluid into a vertical axisymmetric capillary a meniscus is formed at the surface of the liquid column. On the one hand (since the perimeter of the meniscus represents the line of contact for three phases, while this state is characteristic only of one end of the liquid column), we have the possibility of the development of a capillary wave having a length $\lambda_\sigma = 2\pi(\sigma/\Delta\rho g)^{1/2}$ [2]. Since this wave moves along the surface of the capillary, and the thickness r of the liquid layer is commensurate with the length of the wave, its velocity of motion will be $w_\sigma = 2\pi[\Delta\sigma r/(\rho_1 + \rho_2)]^{1/2}/\lambda_\sigma$ [2], while the velocity vector is directed upward. On the other hand, if the meniscus curvature $R_c = r/\cos\theta$, it may be treated as a capillary wave whose length is equal to the perimeter of the circle passing through the generatrix of the spherical surface of the meniscus. At the initial instant of liquid influx into the capillary, since $\theta = 0$, $R_c \to \infty$ and the velocity of the second capillary wave $w_\sigma = [2\pi\sigma/(\rho_1 + \rho_2)\lambda_\sigma]^{1/2}$ is equal to zero and less than the velocity of the first wave, i.e., the speed with which the liquid rises in the capillary is determined by the velocity of the first capillary wave. As the meniscus is formed θ and R_c diminish, the velocity of the second capillary wave increases, reaching and exceeding the velocity of the first capillary wave when $(\sigma/\Delta\rho g)^{1/2}(\cos\theta)^{1/2} \geq r$, i.e., on conclusion of the meniscus formation and satisfaction of the conditions of capillary influx. From this instant on, the velocity of the second capillary wave determines the influx process.

As soon as a liquid column of height h appears in the capillary, this column plays the role of a gravitational wave of length λ_g, equal to the perimeter of a circle of diameter h. Since the vector of velocity for the motion of the gravitational wave $w_g = [g\lambda_g(\rho_1 - \rho_2)/2\pi(\rho_1 + \rho_2)]^{1/2}$ [2] is directed downward, the resulting velocity of liquid motion in the capillary is $w_h = w_\sigma - w_g$. Thus, at the stage of meniscus formation the rate of liquid ascent w_h in the capillary is equal to the difference between the velocities of motion for a capillary wave of length $\lambda_\sigma = 2\pi[\Delta\sigma/(\rho_1 - \rho_2)g]^{1/2}$ and w_g for a gravitational wave of length $\lambda_g = \pi h$ [2]:

$$w_h \equiv \partial h/\partial\tau = 2\pi[r\Delta\sigma/(\rho_1 + \rho_2)]^{1/2}/\lambda_\sigma - [g\lambda_g(\rho_1 - \rho_2)/2\pi(\rho_1 + \rho_2)]^{1/2}. \tag{1}$$

When we integrate expression (1) in limits of $\tau = 0$, $h = 0$ and $\tau = \tau_m$, $h = h_m$, after transformation, we find that the duration of meniscus formation amounts to

$$\tau_m = \left(\frac{\rho_1 - \rho_2}{\rho_1 + \rho_2}\right)^{1/2}\left(\frac{r}{g}\right)^{1/2}\left[2\ln\frac{1}{1 - \left(\dfrac{1 - \sin\theta}{2\cos\theta}\right)^{1/2}} - (2)^{1/2}\left(\frac{1 - \sin\theta}{\cos\theta}\right)^{1/2}\right]. \tag{2}$$

After formation of a meniscus with a curvature radius of $R_c = r/\cos\theta$ the velocity of motion for the inviscid liquid becomes equal to the difference between the velocity w_σ for a capillary wave of length $\lambda_\sigma = 2\pi R_c$ and w_g for a gravitational wave of length $\lambda_g = \pi h$:

Dnepropetrovsk Metallurgical Institute. Translated from Inzhenerno-Fizicheskii Zhurnal, Vol. 60, No. 3, pp. 428-432, March, 1991. Original article submitted February 7, 1990.

0022-0841/91/6003-0338$12.50

$$w_h \equiv \partial h/\partial\tau = [2\pi\sigma/(\rho_1+\rho_2)\lambda_\sigma]^{1/2} - [g\lambda_g(\rho_1-\rho_2)/2\pi(\rho_1+\rho_2)]^{1/2}. \quad (3)$$

Having integrated expression (3) in limits of $\tau = 0$, h = 0 and $\tau = \tau$, h = h, after transformation, we obtain the solution for the time-change in the influx height for the inviscid liquid in the capillary in implicit form:

$$(2)^2\left[\frac{\sigma\cos\theta}{r(\rho_1+\rho_2)}\right]^{1/2}\frac{(\rho_1+\rho_2)}{g(\rho_1-\rho_2)}\ln\left\langle\left[\frac{\sigma\cos\theta}{r(\rho_1+\rho_2)}\right]^{1/2}\Big/\left\{\left[\frac{\sigma\cos\theta}{r(\rho_1+\rho_2)}\right]^{1/2}-\right.\right.$$
$$\left.\left.-\left[\frac{g}{2}\frac{(\rho_1-\rho_2)}{(\rho_1+\rho_2)}\right]^{1/2}(h)^{1/2}\right\}\right\rangle-\frac{(2)^{3/2}h^{1/2}(\rho_1+\rho_2)^{1/2}}{[g(\rho_1-\rho_2)]^{1/2}}=\tau. \quad (4)$$

As $\tau \to \infty$, we obtain an expression for the maximum height of liquid ascent in the vertical capillary:

$$h \to h_{\max} = 2\sigma\cos\theta/gr(\rho_1-\rho_2), \quad (5)$$

known as the Jurin equation.

In the absence of gravity (g = 0)

$$h = [\sigma\cos\theta/(\rho_1-\rho_2)r]^{1/2}\tau, \quad (6)$$

while with gravitational acceleration $a = k_a g$

$$h \to h_{\max} = 2\sigma\cos\theta/k_a g(\rho_1-\rho_2)r. \quad (7)$$

If the capillary is formed by two parallel vertical plates, the distance between these plates being equal to b, then, using the same considerations and bearing in mind that in this case the velocity of the gravitational wave is equal to $w_g = (gh)^{1/2}[(\rho_1-\rho_2)^{1/2}/(\rho_1+\rho_2)^{1/2}]$ [2], we derive an expression for the change in the height of ascent for the inviscid liquidin the capillary over time in implicit form:

$$\left[\frac{(2)^3\sigma\cos\theta}{bg^2}\frac{(\rho_1+\rho_2)}{(\rho_1-\rho_2)^2}\right]^{1/2}\ln\left\langle\left[\frac{2\sigma\cos\theta}{b(\rho_1+\rho_2)}\right]^{1/2}\Big/\left\{\left[\frac{2\sigma\cos\theta}{b(\rho_1+\rho_2)}\right]^{1/2}-\right.\right.$$
$$\left.\left.-(gh)^{1/2}\left[\frac{(\rho_1-\rho_2)}{(\rho_1+\rho_2)}\right]^{1/2}\right\}\right\rangle-2\left[\frac{h(\rho_1+\rho_2)}{g(\rho_1-\rho_2)}\right]^{1/2}=\tau, \quad (8)$$

while the maximum height of liquid influx as $\tau \to \infty$ is given by

$$h \to h_{\max} = 2\sigma\cos\theta/gb(\rho_1-\rho)_2. \quad (9)$$

When an infinite volume of an inviscid liquid is in contact with the vertical wall, the rate of ascent is governed by the difference between the velocity of a capillary wave of length $\lambda_\sigma = 2\pi h(1-\sin\theta)$ and that of a gravitational wave of length $\lambda_g = \pi h$, while the change in the level of ascent for the inviscid liquid along the wall, governed by the capillarity phenomenon, is described by the expression

$$[(2)^3(1-\sin\theta)]^{1/4}[\sigma/(\rho_1-\rho_2)g^3]^{1/4}\times$$
$$\times\,\mathrm{nl}\left\{\frac{[2\sigma(1-\sin\theta)^{1/4}+[gh^2(\rho_1-\rho_2)]^{1/4}}{[2\sigma(1-\sin\theta)]^{1/4}-[gh^2(\rho_1-\rho_2)]^{1/4}}\right\}-$$
$$-[(2)^3h(\rho_1+\rho_2)/g(\rho_1-\rho_2)]=\tau, \quad (10)$$

and as $\tau \to \infty$ the maximum height of ascent amounts to

$$h \to h_{\max} = [2\sigma(1-\sin\theta)/(\rho_1-\rho_2)g]^{1/2}. \quad (11)$$

The spreading out of a droplet of an inviscid liquid over a solid substrate is usually regarded either as free spreading, or as the directed spreading within the confines of a path having a width s.

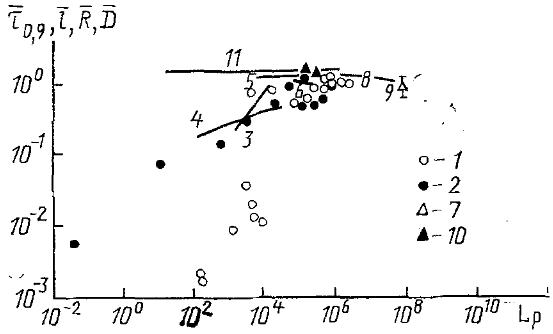

Fig. 1. Comparison of theoretical and experimental discharge parameters: 1) lifting of liquid in vertical capillary [3-7]; 2) inflow of liquid into horizontal capillary [16, 17]; 3) thread of droplet along path [10]; free spreading of droplet along solid substrate; 4) Hg—Zn [8-10]; 5) Ti—graphite [11]; 6) Zr—graphite [12]; 7) steel—W_2C—WC [13]; 8) Si—graphite [14]; 9) Sn—graphite [14]; 10) H_2O—glass [15]; 11) droplet liquid discharge [18].

The free spread of a droplet of an inviscid liquid over a substrate proceeds under conditions in which there exists a line of contact for the three phases. This is the reason behind the appearance of capillary waves whose length is obviously equal to the length of the contact line, i.e., $\lambda_\sigma = 2\pi R$. Since the thickness of the spreading layer is commensurate with the wavelength, then, according to [2]:

$$w_\sigma \equiv \partial R/\partial\tau = 2\pi\,[\Delta\sigma\delta/(\rho_1 + \rho_2)]^{1/2}/\lambda_\sigma, \tag{12}$$

where the thickness of the layer at the start of the spreading amounts to $\delta \approx 2R_0$, while subsequent to the layer assuming the shape of a plane cylinder the thickness of that layer is $-\delta \approx (4R_0^3/3R^2)$. Having solved (12), for the initial period of spreading we obtain

$$R \approx (2)^{3/4} R_0^{1/4} \Delta\sigma^{1/4}\tau^{1/2}/(\rho_1 + \rho_2)^{1/4}, \tag{13}$$

while for the subsequent period we have

$$R \approx (2)^{1/3}(3)^{1/6} R_0^{1/2} \Delta\sigma^{1/6}\tau^{1/3}/(\rho_1 + \rho_2)^{1/6}. \tag{14}$$

This spreading out of the droplet of the inviscid liquid along the path is accomplished under conditions of contact at both ends of the three-phase spread length, which produces the onset of the capillary waves whose length is obviously equal to the perimeter of a circle with a diameter equal to the length of the spreading liquid, i.e., $\lambda_\sigma = 2\pi l$, where l represents the spreading length in one of the directions away from the point of spreading onset. The rate of spreading can be described by expression (12). At the beginning we have $\delta \approx 2R_0$, and for this period the change in the spreading length over time from the center amounts to

$$l \approx (2)^{3/4} R_0^{1/4} \Delta\sigma^{1/4}\tau^{1/2}/(\rho_1 + \rho_2)^{1/4}. \tag{15}$$

After the settling out of the droplet $\delta \approx 2\pi R_0^3/3sl$, and the change, over time, of the spreading length will be defined by the expression

$$l \approx (5)^{2/5}\pi^{1/5} R_0^{3/5} \Delta\sigma^{1/5}\tau^{2/5}/(2)^{1/5}(3)^{1/5}s^{1/5}(\rho_1 + \rho_2)^{1/5}. \tag{16}$$

On influx of an inviscid liquid into a horizontal capillary the velocity of inflow is described by expression (12) for δ = r and $\lambda_\sigma = 2\pi[\Delta\sigma/(\rho_1 - \rho_2)g]^{1/2}$. As a result of the solution we find that the duration of the inflow changes over time in accordance with the relationship

$$l = (gr)^{1/2}(\rho_1 - \rho_2)^{1/2}\tau/(\rho_1 + \rho_2)^{1/2}. \tag{17}$$

The found relationships were used in analyzing the experimental data derived in the rise of the liquid through the vertical capillaries [3-7], the spreading out of the liquid droplets, including melts, over solid substrates [8-15], and the influx of the liquid into horizontal capillaries [16, 17]. Based on analytical expressions from the wave theory of liquid flow, the calculated values were compared with those actually derived. For the ascent of the liquid in a vertical capillary in a gravitational field we calculated the ratio $\bar{\tau}_{0.9}$ between the theoretical and actually durations of ascent to the height $h = 0.9h_{max}$, and at $g = 0$ we calculated the same ratio for fixed levels of ascent. In droplet spreading and in flow into the horizontal capillary we determined the $\bar{R}$, $\bar{l}$ ratio of the actual and theoretical quantities. These characteristics are shown in Fig. 1 as functions of the Laplace criterion. For the ascent of the liquid in the vertical capillary the experimental data are very close to the theoretical for $Lp \geq 10^4$, while for $Lp \geq 10^6$ the wave processes determine the flow of the liquid. In the experiment this is achieved under conditions of no gravitation [3]. Thus, the wave processes predominate for conventional liquids as they flow into capillaries with diameters on the order of 10^0 cm. The capillarity in this case makes itself apparent on a noticeable scale as $g \to 0$.

The flow of the liquid into the horizontal capillary follows quantitative relationships that are quite close to the theoretical for the case in which $Lp \gtrsim 10^5-10^6$. The free spreading of the droplet over the path, including during the initial period of the spreading [13-15], is defined by the wave processes for the case in which $Lp \gtrsim 10^4-10^5$.

Thus, in the one-dimensional motion of the liquid in capillaries it may be regarded as an inviscid fluid and the wave processes become decisive at $Lp \geq 10^6$. In the case of two-dimensional droplet spreading this limit drops to $Lp \approx 10^5$.

It is interesting to note that in the case of three-dimensional motion of the liquid, as it is discharged in the form of droplets out of a nozzle [18], the ratio $\bar{D}$ for the actual and theoretical droplet diameters, under an assumption of a wave-like nature for the process [19], is nearly 1 and constant over a broad range $Lp = 10^1$-10^6.

The slight scattering in data which we can see in Fig. 1 is, apparently, associated primarily with inaccuracies in determining the properties of the liquid, in particular the spreading factor $\Delta\sigma$.

High values for the Laplace criterion can be achieved when the moving liquid exhibits limited viscosity as is the case, for example, with liquefied gases. The experimental data derived in [16] for the influx of liquid oxygen into a horizontal capillary matched these derived quantitative relationships. It might be assumed that the motion of a superfluid liquid, generated by surface phenomena, will be described by analytical expressions such as those derived on the basis of wave theory. Obviously, the use of expressions based on the quantitative relationships governing viscous flow for the description of the hydrodynamics of surface phenomena under conditions in which the liquid may be regarded as inviscid are not acceptable.

CONCLUSIONS

1. We have derived expressions on the basis of wave theory to describe the hydrodynamics of certain surface phenomena.
2. We have demonstrated that under specific conditions a liquid may be regarded as inviscid, and experimental data in such cases are in good agreement with calculations based on the derived expressions.

NOTATION

r and d, radius and diameter of capillaries; R_c, radius of meniscus curvature; θ, edge wetting angle; h, height of liquid ascent; l, spreading (influx) length; R_0 and R, initial and instantaneous radii of free spreading droplet; λ, wavelength; w, velocity; τ, time; ρ, density; μ, viscosity; σ, surface tension; g, gravitational acceleration; a, acceleration; b, width of slotted capillary; s, width of spreading path; δ, thickness of the spreading liquid layer; $Lp \equiv \rho_1 d(\delta)\sigma/\mu_1^2$, Laplace criterion. Subscripts: 1, moving liquid; 2, ambient medium.

LITERATURE CITED

1. B.D. Summ and Yu. V. Goryunov, *Physicochemical Fundamentals of Wetting and Spreading* [in Russian], Moscow (1976).
2. H. Lamb, *Hydrodynamics* [Russian translation], Moscow—Leningrad (1947).
3. R. Siegel, *J. Appl. Mech.*, Ser. E, **28**, No. 2, 165-170 (1961).
4. V. V. Khlynov, Yu. I. Terskov, E. L. Furman, et al., *Zh. Fiz. Khim.*, **53**, No. 7, 1830-1932 (1979).
5. W. E. Brittin, *J. Appl. Phys.*, **17**, No. 1, 37-44 (1946).
6. J. R. Ligenza and R. B. Bernstein, *J. Am. Chem. Soc.*, **73**, No. 10, 4636-4638 (1951).
7. E. V. Gribanova and L. I. Molchanova, *Kolloid. Zh.*, **40**, No. 2, 217-223 (1978).
8. B. D. Summ, Yu. V. Goryunov, N. V. Pertsov, and E. D. Shchukin, *Dokl. Akad. Nauk SSSR*, **137**, No. 6, 1413-1415 (1961).
9. E. V. Goryunov, N. V. Pertsov, B. D. Summ, and E. D. Shchukin, *Dokl. Akad. Nauk SSSR*, **146**, No. 3, 638-641 (1962).
10. E. D. Shchukin, Yu. V. Goryunov, G. I. Den'shchikova, et al., *Kolloid. Zh.*, **25**, No. 1, 108-114 (1962).
11. V. P. Elyutin, V. I. Kostikov, and M. A. Maurakh, *Izv. Vuz. Chem. Metallurg.*, No. 11, 5-10 (1964).
12. V. I. Kostikov and B. D. Summ, *Fiz. Khim. Obrab. Material.*, No. 2, 149-161 (1971).
13. V. I. Shiimov, V. V. Khlynov, and O. A. Esin, *Zh. Fiz. Khim.*, No. 10, 2628-2630 (1968).
14. Yu. V. Naidich, V. M. Perevertailo, and G. M. Nevodni, *Poroshk. Metal.*, No. 7, 51-55 (1972).

15. S. V. Stenovskii, *Zh. Prikl. Mekh. Tekh. Fiz.*, No. 1, 89-92 (1979).
16. J.F. Bitten and E. G. Fochtman, *AIChE J.*, **9**, No. 2, 279-282 (1963).
17. G. I. Den'shchikova, Yu. V. Goryunov, and B. D. Summ, *Vestnik MGU, Ser. 2, Khim.* **18**, No. 4, 419-422 (1977).
18. N. I. Smirnov and S. E. Polyuta, *Zh. Prikl. Khim.*, **21**, No. 11, 1137-1146 (1948).
19. V. B. Okhotskii, *Inzh.-Fiz. Zh.*, **54**, No. 2, 203-211 (1988).

NUMERICAL AND PHYSICAL SIMULATION OF AXISYMMETRIC STREAMLINING OF A STAGED CYLINDER WITH A LOW-VELOCITY FLOW OF AIR

S. A. Isaev, V. M. Suprun, and O. A. Shul'zhenko

UDC 532.517.4

The mechanism to reduce the frontal resistance to the motion of a staged cylinder of circular cross section and limited length is analyzed on the basis of a numerical solution for the Reynolds equations closed by means of a dissipative two-parameter model of turbulence, and through systematic measurement in wind tunnels.

1. Achieving organized or predictable flow separation in the vicinity of streamlined surfaces, from the standpoint of conceptual aerohydrodynamics, is possible in a variety of ways, some of which are discussed in [1]. There is no doubt that the multiplicity of possible situations does not exhaust the material contained in [1], all the more so because primary attention is devoted to an examination of long bodies of revolution. At the same time, practical requirements dictate a broadening of the spectrum of geometric configurations in bodies and, in particular, call for an analysis of the streamlining of short bodies and of bodies that are not circular in lateral cross section, examples of which can be found among the containers used in aviation, maritime, and railroad transport, trailers used in trucking, floating drilling platforms, and the like (see, for example, [2]).

The frontal resistance in poorly streamlined bodies is achieved by locating protrustions of various shapes across the surfaces of these bodies, or by positioning these bodies in the near wake, behind other bodies. In the present study the mechanism used to reduce resistance of bodies during the formation of the leading separation zone, as analyzed in detail in [1] for a cylinder with a protruding disk, using the examples of bodies with considerably simpler geometry, we examine a staged cylinder of circular lateral cross section and of limited length. We should note that the aerodynamics of short cylinders with coaxially positioned disks in the leading and rear areas is presented in [3].

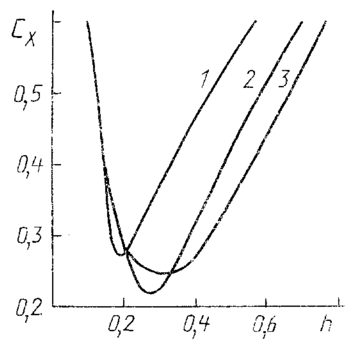

Fig. 1. Experimental functions relating the coefficient of frontal resistance C_x for a stepped cylinder to the step height h for elongations of the protrusion in the step portion of l = 0.2 (1), 0.4 (2), 0.6 (3).

Academy of Civil Aviation, Leningrad. Translated from Inzhenerno-Fizicheskii Zhurnal, Vol. 60, No. 3, pp. 433-439, March, 1991. Original article submitted March 14, 1990.

